

5. The Programming Language Prolog

developed by Kowalski + Colmerauer in the 1970s

Fixes certain syntax rules:

- $:-$, $?-$, ...
- function + predicate symbols: strings starting with lower-case symbol, strings of special symbols ($\langle - \rangle$), strings in quotes 'X'
- variables: strings starting with upper-case symbols or with $_$ ($_G192$)
- special anonymous variable $_$ ("don't care")
 - each occurrence of $_$ can be instantiated differently
 - instantiations of $_$ are not contained in the answer subst.

Ex: Prolog contains the fact $p(a,b,c)$.

$?- p(, , X)$.

$X = c$

- Prolog allows overloading of pred./fact. symbols.

$p(a,b,c)$.

$p/3$

2 unrelated

$p(d,e)$.

$p/2$

pred. symbols

↑

one often writes pred./fact. symbols

in this way to distinguish
pred./fct. symbols of different
arities.

- Prolog uses a variant of unification without
occur check to improve efficiency.

$\text{equal}(X, X).$

? - $\text{equal}(f(Y), g(Y)).$

false

clash failure

? - $\text{equal}(f(0), f(Y)).$

$Y=0$

? - $\text{equal}(Y, f(Y)).$

$Y = f(Y)$

succeeds in Prolog
although one
should have
occur failure

Alternative:

$Y = f(*)$

$Y = f(f(...))$

Solution: $Y / f(f(...))$

↑
infinite term

In general, one should avoid Prolog programs where
possible occur failures could happen and where infinite
terms are constructed.

→ 

Prolog has a pre-defined predicate for proper unification:

? - $\text{unify_with_occurs_check}(Y, f(Y)).$

false

Now we will introduce several features of Prolog

that go beyond pure logic programming.

Sect 5.1 + 5.2: typical pre-defined data types

5.1. Arithmetic

Prolog has no data types, but it only operates on terms.

⇒ Data objects have to be represented by terms.

Ex: \mathbb{N} can be represented by terms over $0 \in \Sigma_0$

and $s \in \Sigma_1$:

$\text{add}(X, 0, X).$

$\text{add}(X, s(Y), s(Z)) :- \text{add}(X, Y, Z).$

Can be used
for addition,
subtraction, ...:

?- $\text{add}(X, s(0), s(s(0)))$

Disadvantage: inefficient and hard to read

3-directionality

$1000 \hat{=} \underbrace{s(s(\dots s(0)))}_{1000 \text{ times}}$

Prolog has built-in arithmetic that allows us to write numbers as usual and to use efficient arithmetic operations provided by the operating system.

Arithmetic expression: term built from numbers, variables,

binary infix symbols $+$, $-$, $*$, $//$, $**$, ...

unary negation $-$

↑ integer division
↑ exponentiation

In principle, these are terms as usual.

$\text{equal}(X, X).$

?- $\text{equal}(3, 1+2).$

false

$+$, $-$, ...
are syntactic
|| - ...

false

? - equal (Y, 1+2).

Y = 1+2.

are syntactic
fct. symbols
that are not
evaluated in
syntactic unification

There exist special pre-defined predicates which
evaluate arithmetic expressions: $<$, $>$, $=$, $<=$, $>=$,

$=$ \neq , \neq \neq
↑ ↑
equality non-equality

For an operation op like this:

? - t_1 op t_2 .

When evaluating this query, t_1 and t_2 must be
fully instantiated arithmetic expressions. Then

the pre-defined fcts $+$, $-$, ... are evaluated
and the boolean result of the comparison
determines whether the query succeeds.

? - $1 < 2$.

true

? - $1 * 1 < 1 + 1$.

true

? - $-2 > 1$.

false

? - $X > 1$.

error

? - monika > 1

error

These predicates cannot be used to instantiate variables.

?- X ::= 2.
error

Therefore, there is another predicate is/2.

?- t₁ is t₂.

When evaluating the query, t₂ must be a fully instantiated arith. expr. Afterwards, t₁ is unified with the result of evaluating t₂.

?- 2 is 1+1. ?- 1+1 is 2.
true false

?- X is 1+1. ?- X+1 is 1+1.
X=2 false

?- X is 3+4, Y is X+1.
X=7, Y=8

?- Y is X+1, X is 3+4.
error

Prolog has several predicates for equality:

- ::= arithmetic equality where both arguments are evaluated

- is equality, where the right argument is evaluated and afterwards one performs unification
- = unification (corresponds to $\text{equal}(X, X)$)
without occur check

? - $\text{monika} = \text{monika}$
true

? - $f(X) = f(a)$
 $X = a$

? - $X = 1 + 1$
 $X = 1 + 1$

? - $1 + 1 = 2$
false

? - $f(X) = X$
 $X = f(X)$

- $\text{unify_with_occurs_check}$

- == syntactic equality

? - $\text{monika} == \text{monika}$
true

? - $f(X) == f(Y)$
false

Computing with arithmetic:
 $\text{add}(X, 0, X)$

without
built-in

$add(X, 0, X).$

$add(X, s(Y), s(Z)) :- add(X, Y, Z).$

without
built-in
arithmetic

Instead:

$add'(X, 0, X).$

$add'(X, Y+1, Z+1) :- add'(X, Y, Z).$

Disadvantage: $?- add'(1, 2, X).$
false

$?- add'(1, 0+1, X).$

$X = 1+1$

Better:

$add''(X, 0, X).$

$add''(X, Y, Z) :- Y > 0, Y1 \text{ is } Y-1,$

$add''(X, Y1, Z1), Z \text{ is } Z1+1.$

$?- add''(1, 2, X).$

$X = 3$

$?- add''(X, 2, 3).$

error



because at some point one
has to evaluate an is-literal
where the right argument is
not fully instantiated

⇒ We lose bidirectionality.

Easier: $\text{add}(X, Y, Z) :- Z \text{ is } X+Y.$
and
much
more
efficient

Ex: gcd on natural numbers

$\text{gcd}(X, 0, X).$

$\text{gcd}(0, X, X).$

$\text{gcd}(X, Y, Z) :- X \leq Y, X > 0, Y_1 \text{ is } Y-X,$
 $\text{gcd}(X, Y_1, Z).$

$\text{gcd}(X, Y, Z) :- X > Y, Y > 0, X_1 \text{ is } X-Y,$
 $\text{gcd}(X_1, Y, Z).$

? - $\text{gcd}(28, 36, X).$

$X = 4$

Again, this is not bidirectional. To implement bidirectional arithmetic programs: Constraint Logic Programming (CLP)

Pre-defined pred. $\text{number}/1$ is true if

the argument is a number when the pred. is evaluated:

? - number (2).

true

? - number (1+1).

false

? - X is 1+1, number (X).

X=2

? - number (X).

false